Multi-Coset Sampling based Wideband Spectrum Sensing in Cognitive Radio using Greedy Algorithm

*Deepak Papneja, ** Dr. Munish Rattan and *** Prof. Jasmeet Kaur
*M. Tech.,GNDEC, Ludhiana
Deepak.purc@gmail.com
**Assistant Professor, GNDEC, Ludhiana
Dr.munishrattan@gmail.com
***Assistant Professor, GNDEC, Ludhiana
Jasmeetkr.90@gmail.com

Abstract: Spectrum sensing is one of the predominant function of cognitive radio technology where the requirement of a high sampling rate in the sensing of a wideband signal is a challenging issue. In fact, the main problem associated with such wideband spectrum sensing process is that it is either impractical or too expensive to exhibit Nyquist sampling on such signal because of need of complex Analog to Digital converters. Thus, considering these factors, a spectrum sensing scheme using multicoset based sub-Nyquist sampling paradigm has been developed in this research paper. The prime objective of this work is to develop a scheme for spectrum sensing in cognitive radio without using more analog to digital converter or RF front end converters. Unlike other conventional approaches of spectrum sensing, where initially the wideband signals are regenerated from the sub-Nyquist samples, and then the power estimation takes place, here in this developed paradigm, the power spectrum of the wideband signal has been sensed directly using statistical characteristics. This has strengthened it for saving huge sampling rates and time for cognitive radio sensing having multiple sub-bands. The enhanced multicoset based sampling and later the spectrum estimation and energy detection of the cosets signals using greedy algorithm has strengthened the proposed system to deliver optimal results in terms of universal employment, minimal cosets, location of active bands etc.

Keywords: Cognitive Radio, Wideband Spectrum sensing, Sub-Nyquist sampling, Correlation matrix, Subspace analysis, Greedy Algorithm.

Introduction

Under an analysis that the average spectrum occupancy is around 5%, for the purpose of improving the spectrum utilization efficiency and providing high bandwidth to mobile users, the next generation communication networks [1] program was developed to implement spectrum policy intelligent radios, also known as Cognitive Radios [2], by dynamic spectrum access techniques as shown in Figure 1. To exploit spectrum opportunities, cognitive radio must detect spectrum holes. Traditional spectrum sensing algorithms can then be used for searching spectrum holes. Majority of the existing sensing techniques such as energy detection, filter bank spectrum sensing and multi-taper spectrum estimation techniques etc, functions using Nyquist rate sampling concept. In the wideband regime, a major challenge arises from the stringent requirements by the high

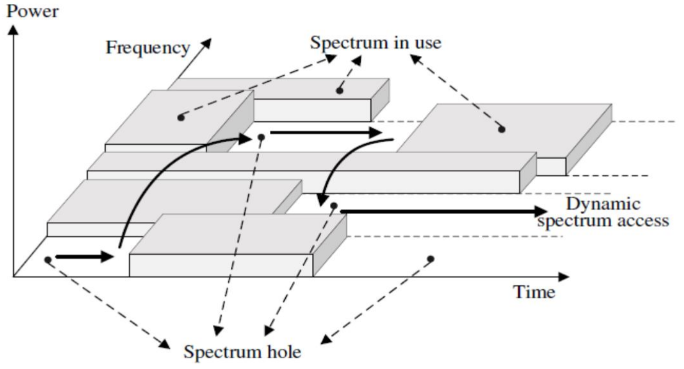

Figure 1: Illustration of spectrum holes and the concept of dynamic spectrum access [3]

sampling rate on the analog-to-digital (A/D) converters employed in the receiver. Sub-Nyquist sampling, which is also stated as the compressive sampling represents the approach to recover signals from spectrum sample retrieved by means of a rate below the Nyquist criteria. One generic paradigm for wideband sensing based on sub-Nyquist sampling **is a two-phase technique**. In this approach, at first the wideband spectrum signal is reconstructed by means of the sub-Nyquist samples, which is then followed by spectrum sensing. In this research paper, the two phase (dual phase) spectrum sensing approach with greedy algorithm has been developed.

Related Work / Literature Studies

A number of researches have been proposed for spectrum estimation using sub-Nyquist sampling. Researchers in [4], considered each sub-band with the crisp edges and employed an edge detection approach for sub band detection. In [5], the co-prime sampling approach was proposed that comprises two sampling branches having co-prime sampling rates. Researchers also proposed a frequency-domain approach in [5] where they can measure the power spectrum directly after converting the time-domain samples into frequency domain. Similarly, in [6], a number of multicoset sampling schemes were explored for spectrum sensing. In [7], researchers minimized the computational complexity and reduced the compression ratio to enhance overall performance. Researchers in [8] developed a frequency domain scheme based on the bandwidth and frequency resolution parameters. In [9], researchers employed finite number of noisy samples to estimate the correlation matrix of a which has been further employed by a non-linear least square (NLLS) estimator for detecting the available and busy channels of the spectrum. [10] proposed greedy and optimal approach in CR for the opportunistic spectrum access. The maximum spectrum reuse and minimum interference between primary user (PU) and secondary user (SU) during the opportunistic spectrum usage by SU, has been the main motive of this paper. [11] proposed a sub-Nyquist sampling scheme using modulated wideband converter that performs sampling with more than one branch using relatively low rate ADCs, which is then followed signal recovery using compressive sensing approach. [12] proposed to signal reconstruction system from the power spectrum of signals using sub-Nyquist sampling.

So, a number of approaches have been estimated for spectrum sensing into CR but taking into consideration of optimal system for facilitating better spectrum sensing and resource utilization even ensuring minimal A/D requirements, majority of systems couldn't deliver better results. Even in majority of systems, the channel status and activities couldn't be retrieved which might be significant factor to ensure optimal resource allocation in CR. On the other hand very few works have emphasized on wideband spectrum sensing. Therefore, considering these requirements, here in this research paper, a scheme for spectrum sensing has been developed using Sub-Nyquist sampling paradigm along with greedy algorithm. The proposed system would tend to accomplish the overall objectives to ensure optimal CR performance.

Statement

While the uniform sampling theorem is suitable for low-pass signals and an efficient sampling with minimum rate is attained, it seems quite inefficient in case of signals with multiple bands with sparse spectrum. This comes from the fact that multiband signals have some gaps between each band that tempts one to work with a rate lower than Nyquist rate. So, here in this work, it is proposed to study the periodic non-uniform sampling also called Multi-coset Sampling [13] and reformation of multiband sparse spectrum signals. For this, one hypothesis has been proposed that for each signal element, the signal vector has p known elements while the vector has L unknown elements and as $p < L$, then the number of equations or the computational complexities can be reduced to a great extent and it would be less than the number of unknowns. In the non-uniform sampling, the key parameters are sampling time; cosets etc which are needed to be selected properly for a perfect and optimal reconstruction. The most useful criteria to choose these parameters are minimum sampling rate, minimum error and perfect reconstruction. With an objective to develop a model with minimal sample rate, the large value of sample pattern and minimal cosets p are needed which is in general bound by a factor called Landau lower bound. So, in this proposed work, it is tried to implement or select the value of cosets p with respect to the number of active cells q, with a condition $p \geq q$. And hence, in this proposed work, the well conditioned system so used is defined as a system where even if a small change in the coefficient matrix could cause the variation in the solution vector. To achieve this, it is proposed to use a vector called condition vector here which is needed to be minimal for the coefficient matrix. In major existing approaches, certain exhaustive search algorithms have been employed, but it can be efficient only for small values of samples or cosets and infeasible for large values of L and p. So to optimize this, here it is proposed to reduce the cost factor of search and therefore such an algorithm is developed here that can add or remove feature of patterns in sequences. Majority of existing schemes to achieve this suffers from local minima, thus considering these issues here it is proposed to employ sequential forward search or greedy search approach. Also, to obtain the minimal condition number and consequently, lowest Mean Square Error (MSE), in this proposed research work, a greedy approach or algorithm has been implemented.

Innovative Content

Multi-Coset sampling

In multicoset sampling, at first, a proper sampling period T is selected while maintaining the criteria that it doesn't cause any aliasing effect. It is then followed by sampling of the signal $x(t)$ non-uniformly at the instants $t_i(n) = (nL + c_i)T$ for $1 \leq i \leq p$ and $n \in \mathbb{Z}$.

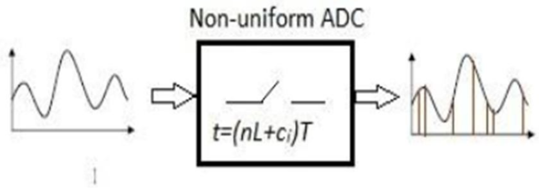

Figure 2. Multi-coset sampling technique [9]

The received analog wideband signal $x(t)$ is sampled using a multicoset sampling scheme that yields p data sequences for $i = 1, \ldots, p$, given by [14]

$$x_i(m) = [x(mL + c_i)/B_{max}], m \in Z \qquad (1)$$

where $\{c_i\}$, is p selected numbers randomly out of the set $L = \{0, 1, \ldots, L - 1\}$.

The mean sample rate thus obtained is $f_{avg} = \alpha B_{max}$, where $\alpha = \frac{p}{L}$is called the sub-Nyquist factor. According to Landau's lower bound, α is lower bounded to the maximum channel occupancy $\alpha \geq \Omega_{max}$ [15].

Correlation matrix

The correlation matrix of sampled data is computed. Firstly, every $x_i(m)$ sequence obtained from equation(3.1) is over-sampled by a factor L, such that

$$x_{u_i}[s] = \begin{cases} x_i\left(\frac{s}{L}\right), & s = mL, m \in Z \\ 0, & otherwise \end{cases} \qquad (2)$$

and after applying the filter, it is obtained

$$x_{h_i}[s] = x_{u_i}[s] * h[s], \qquad (3)$$

where $h[s]$is the interpolation filter whose frequency response is

$$H(f) = \begin{cases} 1, & f \in [0, B] \\ 0, & otherwise \end{cases} \qquad (4)$$

Next, the output sequence which are obtained after filteration is delayed with c_i samples such that

$$x_{c_i}[s] = x_{h_i}[s - c_i] \qquad (5)$$

Let us define a new vector $y(f)$ as the known vector of observations [13]

$$y(f) = [X_{c_1}(f), x_{c_2}(f), \cdots, x_{c_p}(f)]^T \qquad (6)$$

where the notation T is for the transpose, and $x_{c_i}(f)$ shows the DFT of the sequence $X_{c_1}(n)$. Also, $x(f)$, is the vector containing the unknown signal spectrum parameters and is defined as

$$x(f) = \begin{bmatrix} X(f + g_1 B) \\ X(f + g_2 B) \\ \vdots \\ X(f + g_N B) \end{bmatrix}, f \in [0, B] \qquad (7)$$

where $X(f + g_i B), f \in [0, B]$, are the frequency domain elements of the signal in the active band indexed by g_i.

On application of Fourier transform, the results obtained are expressed in matrix form. So, the data model so formed in the frequency domain is given by

$$y(f) = A(g).x(f) + n(f), \quad f \in [0, B] \; (8)$$

where$A(g) \in C^{p \times N}$is the modulation matrix

$$A(g)(i, k) = B \exp\left(\frac{j2\pi c_i g_k}{L}\right) \qquad (9)$$

And $n(f)$ depicts the representation of the noise in the frequency domain. For the general case, it is considered that $n(f)$ is a Gaussian complex noise with distribution of$N(0, \sigma^2 I)$, and this noise is also uncorrelated with the signal.

The correlation matrix for observation vector thus obtained is defined as [14]

$$\begin{aligned} R &= E[y(f).y * (f)] \\ &= A(g).P.A * (g) + \sigma^2 I \qquad (10) \end{aligned}$$

where$()^*$ depicts the Hermitian transpose, and

$$P = E[x(f).x*(f)] \quad (11)$$

is the signal vector's correlation matrix.

Condition Number

Therefore, given $\boldsymbol{y}(f)$ the problem is to estimate the vector $\boldsymbol{k}$ with minimal length q, for some $\boldsymbol{z}(f)$. Considering realistic environment with certain non-idealities that a model can be derived as

$$y(f) = A_C(k)z(f) + n(f) \quad (12)$$

As the signal distribution is unknown and therefore $\boldsymbol{R}$ cannot be retrieved. Then $\boldsymbol{R}$ is estimated from the measured data as

$$R \triangleq \int_0^{f_{max}} y(f)y*(f)df \quad (13)$$

with the dimension of $p \times p$, the ( )* denotes the Hermitian transpose. The correlation matrix can be obtained as

$$R = \int_0^{\frac{f_{max}}{L}} A_c(k)z(f)z*(f)A*_c(k)df$$
$$= A_c(k)zA_C^*(k) \quad (14)$$

where$\mathbf{Z} \geq 0$ is a $q \times q$ matrix given by

$$Z = \int_0^{\frac{f_{max}}{L}} z(f)z*(f)df$$

The condition number of a matrix A is defined as

$$cond\ (A) = \|A\|.\|A^{-1}\|$$
$$= \frac{\sigma_{max}(A)}{\sigma_{min}(A)} \quad (15)$$

where$\|\quad\|$ represents the norm operation and $\sigma_{max}$and $\sigma_{min}$are the maximum and minimum singular values respectively. So this kind of sampling pattern can be obtained as the solution to the minimization problem described below [16]:

$$C_{opt} = arg \min_{C:|C|=p} cond\ (A_C(k)) \quad (16)$$

here the symbol $|\boldsymbol{C}|$ tells the cardinality or length of the set $\boldsymbol{C}$.

Estimation of the Number of Active channels

In this proposed model, the number of active channels has been estimated using MDL criterion given as $0 \leq r \leq N_{max}$ [9]

$$N\ or\ \hat{q} = arg \min_r -M(p-r)\log\frac{g(r)}{a(r)} + \frac{1}{2}r(2p-r)\log M \quad (17)$$

Here Mindicates the total number of samples, and $g(r)$represents the geometric mean of the eigenvalues

$$g(r) = \prod_{i=r+1}^{p} \lambda_i^{\frac{1}{p-r}}$$

And $a(r)$ represents the arithmetic mean of the eigenvalues

$$a(r) = \frac{1}{p-r}\sum_{i=r+1}^{p}\lambda_i$$

Thus, implementing above approach of sensing approach, the number of active channels in cognitive radio environment has been estimated.

Sequential forward selection (SFS) or Greedy Search Algorithm

Sequential forward selection (SFS) is also called simplest greedy search algorithm.

SFS performs the best when the optimal subset has less number of features, e.g. (a) when the search is towards the full set, then the region examined by this algorithm is narrower and (b) when the search is almost near the empty set, where a larger number of states can be evaluated.

With a sample set of $\{0,1,\dots,L-1\}$, consider the intension to retrieve the subset$C = \{c_1, c_2, \dots, c_p\}$, with $p < L$ as described above, that reduces the objective function $cond(\boldsymbol{A_C}(\boldsymbol{k}))$. The SFS approach initiates from the null or empty set and progressively adds the feature c^+, which as a result into minimum value of objective function when it is combined with the set $C_k$which has already been selected [17].

The algorithm for selecting the sample pattern with minimum condition number has been summarized as follows:

1. Initialize C_i with the empty set $C_0 = \{\emptyset\}$
2. Choose the next best future using following equation

$$C^{\dagger} = arg \min_{c \notin C_k}[cond(A_{C_k U_c}(k))]$$

3. update$C_{k+1} = C_k \cup c^{+}; k = k + 1$
4. Repeat step 2 for the condition $k < p$

The search space is retrieved in the form of an ellipse so as to focus on the fact that there are relatively less number of states required towards whether full sets or empty sets.
To perform optimal performance the number of parameters such as p out of L is selected as follows:
No. of comparisons done for the 1st element: L
No. of comparisons done for the 2nd element: L-1
No. of comparisons done for the p-th element: $L - p + 1$
No. of comparisons done for the AP (arithmetic progression) can be obtained using following expression:

$$S_p = pL - \frac{p(p-1)}{2}$$

Location of active slots
After finding the number of active slots, the localization of the active slots has been done as follows:

$$P_{MU}(k) = \frac{\|a(k)\|^2}{\|a^*(k)\hat{E}_n\|^2}, \quad 0 \leq k \leq L-1 \qquad (18)$$

Where k represents the spectral index and $\boldsymbol{a}(k)$ is the *k-th*column of $\boldsymbol{A_C}$, given by

$$a(k) = \frac{1}{LT}\begin{bmatrix} e^{\frac{j2\pi kc_1}{L}} \\ e^{\frac{j2\pi kc_2}{L}} \\ \vdots \\ e^{\frac{j2\pi kc_p}{L}} \end{bmatrix}$$

The above equation () generates L values for L spectral indices such that if k is an active cell, the value of P_{MU} is vital in that point, and otherwise it will be smaller than a threshold. The location of the active slots is then specified by choosing $\hat{k}$ significant values of the computedP_{MU}:

$$\hat{k} = \{k_i | P_{MU}(k) > threshold\} \qquad (19)$$

Thus, implementing above discussed approaches and methodologies, in this research paper, the spectrum sensing has been done for cognitive radio network.

Results And Sensitivity Analysis And Justification Of Results

In this research work, a Matlab based model has been developed. As already discussed above, the Greedy Algorithm has been implemented in this work, so as to obtain Minimum Square Error (MSE) for different cases. So for the different cases obtained after each iteration of greedy algorithm, a series of graphs are obtained (5 here), each case with a different (a) eigen values graph, (b) location of active slots graph, (c) power graph with respect to spectral index, (d) signal spectrum vector graph w.r.t. spectral set, also showing MSE for that particular case, and (e) reconstruction of signal graphs.
The first graph in this series shows that the number of active slots can be estimated from ordered eigenvalues of sample correlation matrix derived from the samples retrieved which is further employed for power or energy estimation. The following figures Figure 3 obtained after 1st iteration, and Figure 4 obtained after 5th iteration illustrate the eigenvalues obtained from correlation matrix derived from samples and multicoset based sampling paradigm. Considering p, sample sequences as developed in program $\mathrm{xhi}[n], i = 1, \ldots, p$ are taken, then the sample correlation matrix has been estimated from all M samples and respectively, eigenvalues have been calculated. The p ordered eigen values, in general $\lambda_1 \geq \cdots \geq \lambda_q \ldots \geq \lambda_p$ and in such cases q eigenvalues becomes significant which ultimately states that (p-q) eigenvalues are existing in noisy circumstances. . Now analyzing these figures, it can be found that in CRN spectrum sensing there exist some noise between some subbands. So, the gap between these
subbands depends on noise and thus, selecting some optimal value of q out of p, the test can be performed for optimal results.
The second figures in this series Figure 5 (obtained after 1st iteration) and Figure 6 (obtained after 5th iteration), represent the location of active slots with respect to the spectral index of the slots. The inclination of various graphs illustrates the activity of the subbands in the cognitive radio network.
The third figures in this series Figure 7 (obtained after 1st iteration) and Figure 8 (obtained after 5th iteration) represents the number of active channels as well as its location in the considered configuration of CRN. From these figures, it can be found that the overall active sub-bands are the spectral sets shown in these figures. Number of cosets or non-linear Sub-Nyquist samples has been obtained as 7, which is minimal and thus it reduces the number of samples as well as sampling time for spectrum sensing. Here different number of channels have been taken into consideration with 3 subbands like 199 channels

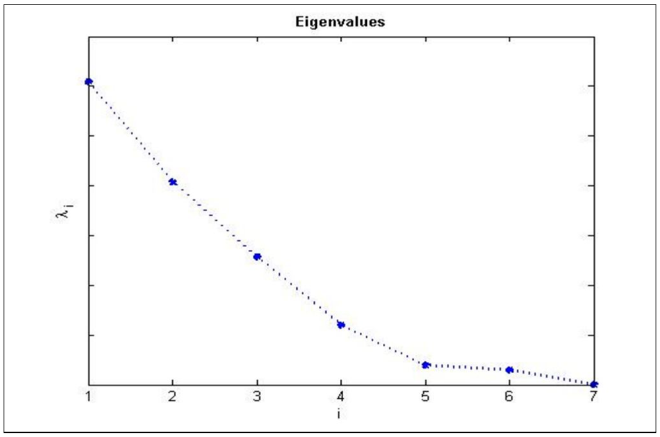

Figure 3. Typical ordered eigenvalues obtained after 1st iteration

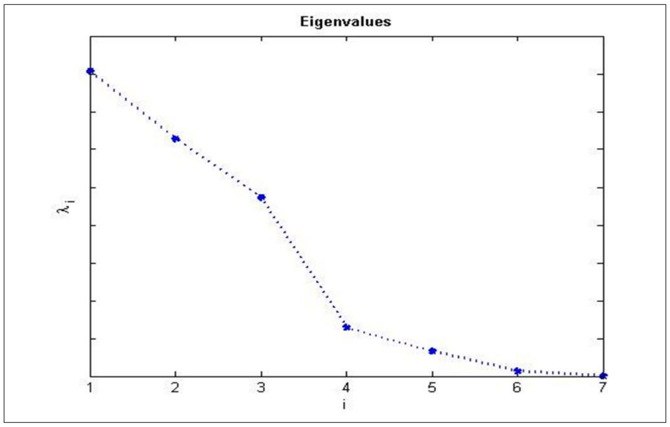

Figure 4. Typical ordered eigenvalues obtained after 5th iteration

in 1st case(figure 7), 28 channels in 5th case(figure 8), 24 channels in 6th case and 15 channels in last case(in 13th iteration) in third series of figures.

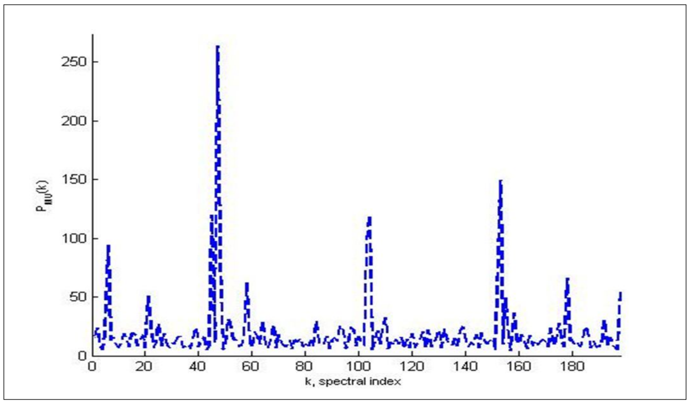

Figure 5. Location of active slots(1st iteration)

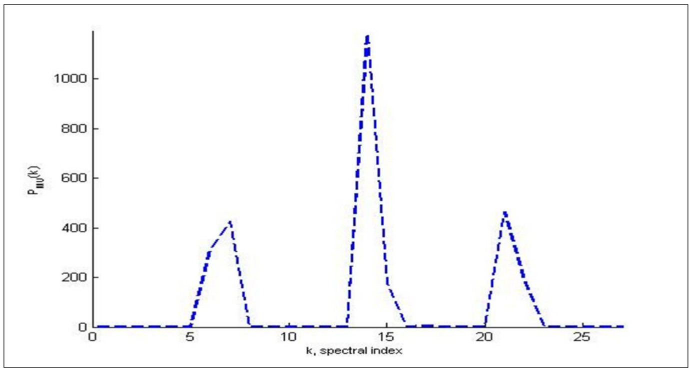

Figure 6. Location of active slots in 5th iteration

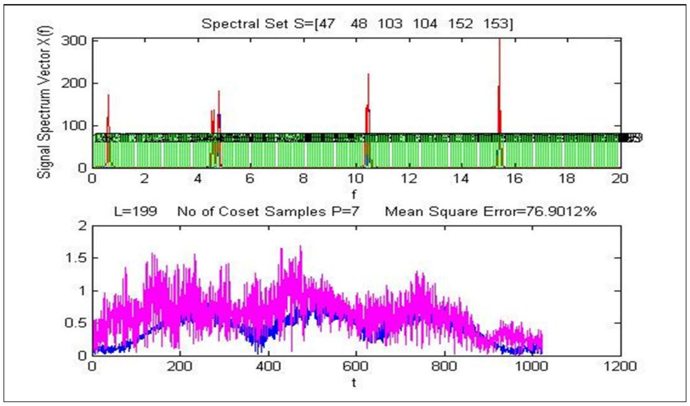

Figure 7. Illustration of active slots or subbands and its location with number of respective samples or cosets in Sub-Nyquist sampling and also showing MSE for 1st iteration

Similarly, in the fourth series of figures, Figure 9 (obtained after 1st iteration) and Figure 10 (obtained after 5th iteration), the respective spectrum strength and signal in frequency bins have been depicted. These figures also illustrate the position of active signal which has been accomplishing using the approach of power spectrum estimation in cognitive radio.

As proposed in this research work, initially the samples using Multicoset sampling based approach of Sub-Nyquist sampling need to be obtained, which is supposed to be followed by the power spectral estimation and then on the basis of power spectral the threshold has been compared. Here the value of PMU is significant, and otherwise it will be smaller than a threshold. The location of the active slots is then specified by choosing significant values of the computed P_{MU}.

$$K_{Act} = \{k_i | P_{MU}(k) > Thrshold\}$$

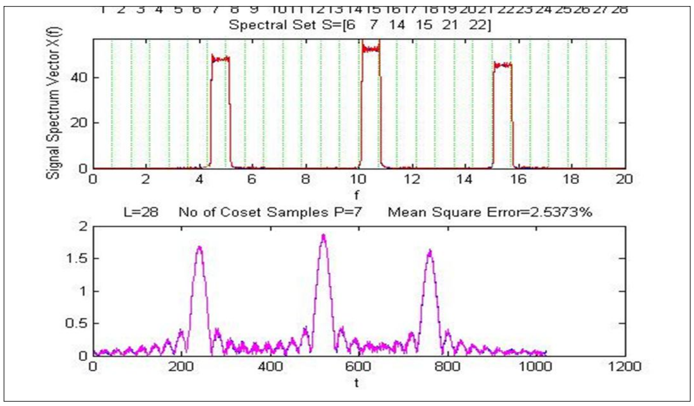

Figure 8. Illustration of active slots or subbands and its location with number of respective samples or cosets in Sub-Nyquist sampling and also showing MSE for 5th iteration.

On the basis of the above threshold based power spectrum comparison the active slots could be identified. Here, it should b noted, that in this work, since the uncertainty in CRN has been taken into consideration, and therefore the estimation of spectral index k, has been done using Multicoset samples

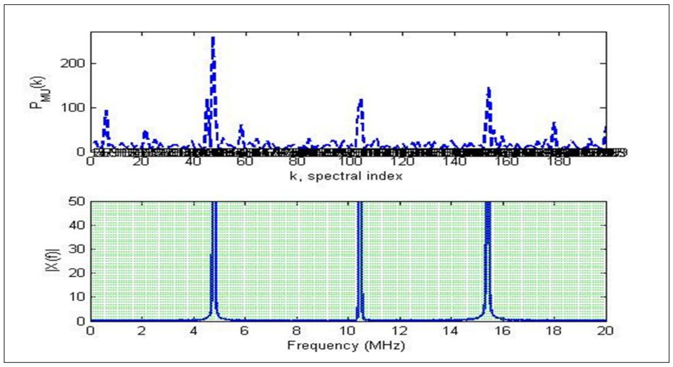

Figure 9. Spectral index based estimate of active subbands or slots in CRN (1st iteration).

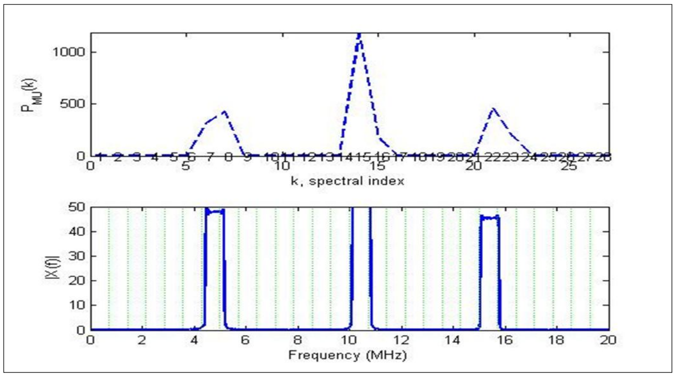

Figure 10. Spectral index based estimate of active subbands or slots in CRN (5th iteration).

The fifth series of figures Figure 11(obtained after 1st iteration) and 12 (obtained after 5th iteration) illustrate the input and reconstructed signals in time and frequency domain. So in this way, the reconstruction of the input signals is also done here.

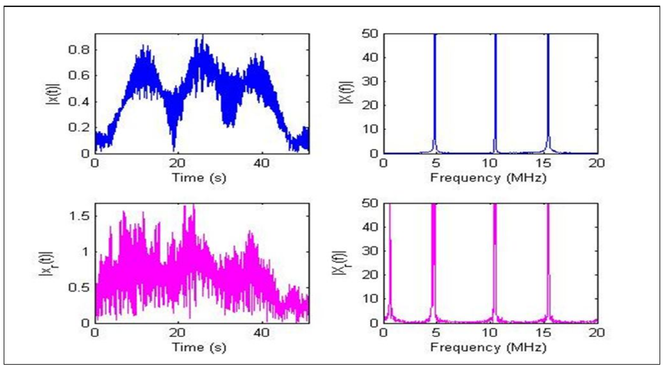

Figure 11. Input signals in the time domain and reconstructed signals in the frequency domain.(1st iteration)

So on each iteration of greedy algorithm, the number of channels and the spectal set changes, correspondingly, the mean square error (MSE) also changes, which turns out to be 76.90% for the first iteration to the 2.53%, that is minimum, for the 5th iteration and going upto the maximum to $9.27*10^{14}$ % for 6th iteration. So as known, the lowest MSE is the optimum solution or the desired outcome, hence, it is considered that the 5th iteration values as the best solution for this research work. And then, the parameters related to this iteration are noted down as the best outcome for future functioning of Cognitive

Radio. Here, the Figure 13. shows the different MSE values obtained after all iterations of greedy algorithm, where red point indicates the minimum MSE value that is 2.53%. The table 1. also shows all the numeric values of MSEs tabulized.

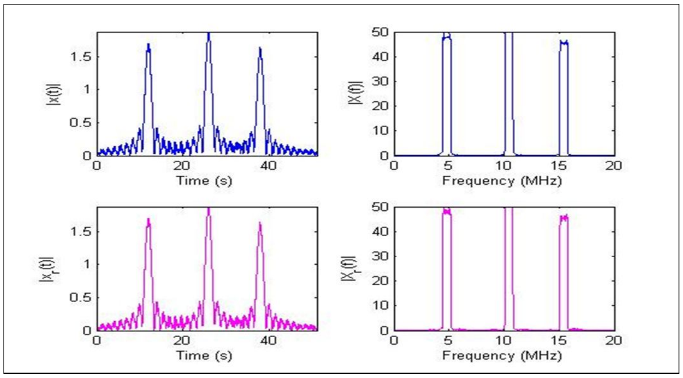

Figure 12. Input signals in the time domain and reconstructed signals in the frequency domain (5th iteration).

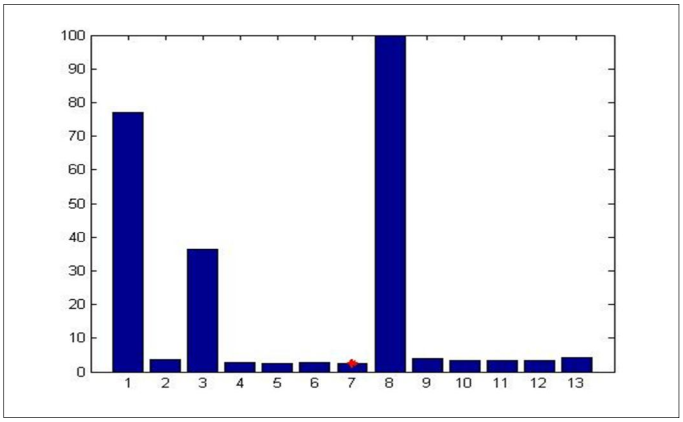

Figure 13. MSE Graph showing different values obtained on each iteration of Greedy Algorithm, and Red point is indicating the minimum MSE value obtained

Table 1. Shows different numeric values of MSE obtained

Iterations	1st Iteration	2nd Iteration	3rd Iteration	4th Iteration	5th Iteration	6th Iteration	7th Iteration
MSE Values	76.90 %	3.65 %	36.30 %	2.77 %	2.61 %	2.66 %	2.53 % (min)

Iterations	8th Iteration	9th Iteration	10th Iteration	11th Iteration	12th Iteration	13th Iteration	
MSE Values	$9.27*10^{14}$ % (max)	3.83 %	3.39 %	3.34 %	3.46 %	4.32 %	

Comparison Of The Results

In table 2, a comparison of the Mean Square Errors or Reconstruction error with the results in literature [18] has been shown. [18] proposed Sub sampling rate subspace estimator approach for sensing of wideband spectrum and found the active slots accordingly with reduced sampling rate. And after finding the channels that represents signal, the reconstruction of the signal has also been done in the above paper. The reconstruction error/ MSE found in [18] is 3.88%, whereas MSE (minimum

MSE) found in this research work is 2.53% which is very much less than the compared literature. So, active slots with reduced sampling rate as well as with less MSE are found in this research work.

Table 2. shows comparison of MSE values obtained in this research work with the latest paper [18]

Sr. No.	Approach Used	Number of frequency channels chosen	MSE obtained
(Singh and Beniwal, 2016)	Sub Sampling Rate Subspace Estimater	3	3.88%
Proposed Work	Greedy Algorithm with Sub space analysis	3	2.53%

Conclusion and Future Scope

Spectrum sensing is considered to be the fundamental process in advanced cognitive radio systems. In this research work, a dual phase Sub-Nyquist sampling and multi-coset sampling based spectrum sensing algorithm has been developed that substantially reduces the sampling rate and computational complexities. The basic method in the developed scheme is comprised of multicoset sampling of the signal, followed further by power spectrum estimation and then by energy detection. In the developed approach of wideband spectrum sensing, the only previous knowledge required is an upper limit on the number of active subbands in the frequency domain of interest. The predominant contribution of this work is the implementation of spectrum sensing in linear as well as non-linear and dynamic cognitive radio network using correlation analysis, subspace method and various other approaches such as greedy approach called Sequential forward search, minimal description length based Eigen method etc. In this work, the proposed scheme not only ensures optimal active spectrum identification but also ensures lowest MSE and minimal number of cosets that as a results affirms minimal interference and aliasing probability. Thus, implementing overall system, a multiband sensing algorithm has been obtained that can significantly estimate the number of active channel in the CRN. It can play significant role or optimal and efficient resource allocation.

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